EEC 4230 - Mobile Communication Systems

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Lecture 8: BER Performance and Capacity of Mobile Radio-I

Outline

1. Capacity of fading channels
2. Digital modulation performance in fading channels
3. Equalization
4. Diversity
5. Channel coding techniques for mobile radio
6. MIMO systems

Introduction

Capacity Analysis [1]

- Study how much data rate is possible over a wireless channel?

Error mitigation

Techniques employed for enhancing error rate performance in wireless communications include:

- Modulation
- Channel Equalization
- Channel coding
- Diversity transmission
- MIMO transmissions, etc.

[1] A. Goldsmith, "Wireless Communications," Cambridge Univ. Press, NY, USA.

Capacity of fading channels

Capacity of AWGN (wireline) channels

- Consider a discrete-time AWGN channel with input/output relationship:

$$y[i] = x[i] + n[i]$$

- Assume channel bandwidth B, and received signal power P. The received SNR is given by $\gamma = \frac{P}{N_0 B}$, where $\frac{N_0}{2}$ is the power spectral density (PSD), or power per Hertz, of the noise.
- The capacity C, for this channel is given by Shannon's formula:

$$C = B \log_2(1 + \gamma) \qquad \text{bits/s (bps)}$$

- Shannon's coding theorem in 1940s proves that a code exist that achieves data rates arbitrarily close to capacity with arbitrarily small error rate. The converse theorem also shows that any code with rate $R > C$ has probability of error bounded away from zero (i.e., impossible). The theorem can be proven using the concept of mutual information between channel input and output.

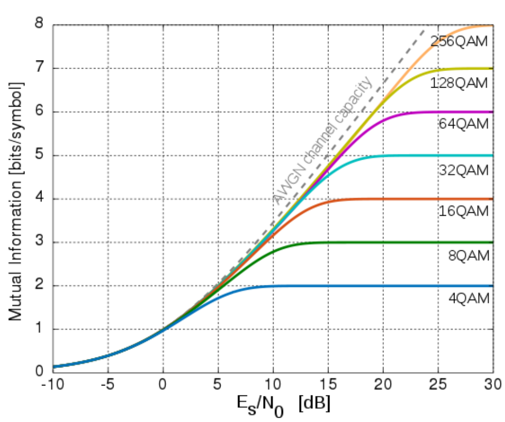
Capacity of fading channels

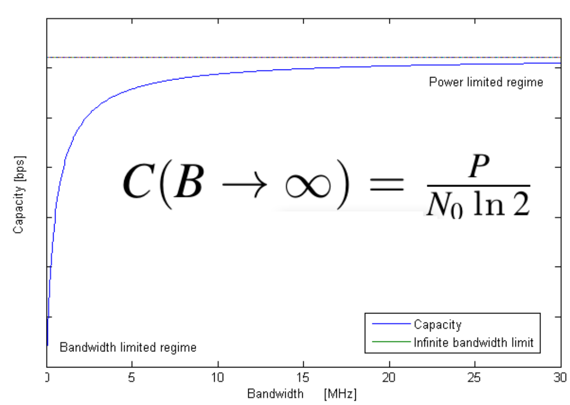
Capacity of AWGN (wireline) channels

- Shannon capacity is generally used as upperbound on the data rates achievable under practical constraints. Typically $R \leq C$ in all practical applications.
- If $R \leq C$, transmitted data can be decoded correctly at receiver
- If $R > C$, transmitted data cannot be decoded correctly at receiver
- E.g. Shannon capacity formula predicts maximum data rate over standard telephone lines as: 30 kbps.
- Advanced hardware designs, modulation and coding techniques have brought the speed of commercial modem of today very close to this (up to 28.8 kbps).
- Note that DSL has higher rate than this, but it uses a different technique than the standard telephone lines.
- Recently turbo codes were designed with data rate just fraction of dB from reaching the Shannon capacity limits [2].

[2] C. Heegard and S. B. Wicker, Turbo Coding, Kluwer, Boston, 1999.

Capacity of fading channels





$$C(B \to \infty) = \frac{P}{N_0 \ln 2}$$

Capacity of fading channels

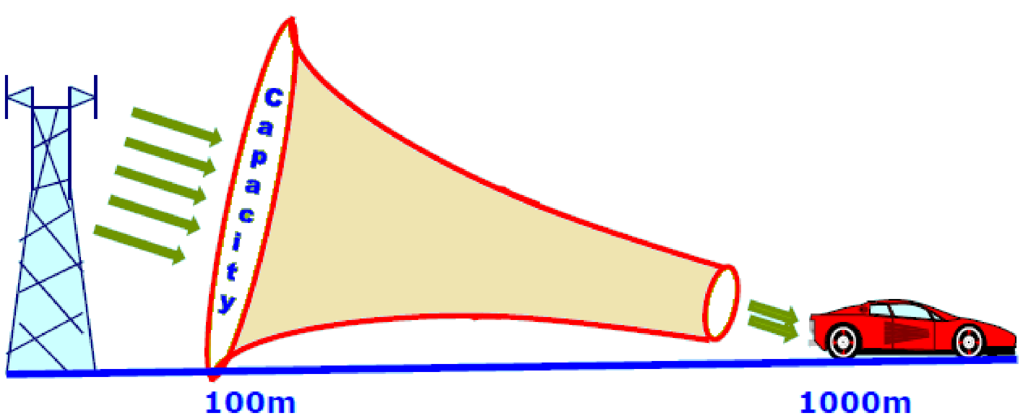
Example: 1

Consider a wireless channel where power fall off with distance follows the formula $P_r(d) = P_0(\frac{d_0}{d})^3$, where $d_0 = 10$m and $P_0 = 1$ W. Assume the channel has bandwidth $B = 30$ kHz and AWGN with noise PSD $N_0/2$, where $N_0 = 10^{-9}$ W/Hz. Find the capacity of this channel for a transmit-receive distance of 100m and 1km.

Solution

- For $d = 100$m: Received SNR is $\gamma = \frac{P_r(d)}{N_0 B} = \frac{(0.1)^3}{(10^{-9} \times 30 \times 10^3)} = 33 => 15$dB. Hence, $C = B \log_2(1 + 33) = 152$ kbps.
- For $d = 1000$m:$\gamma = 0.033 => -15$ dB and the corresponding capacity $C = B \log_2(1 + .033) = 1.4$ kbps.
- Note the Dramatic decrease in wireless capacity C as $d \ggg$. Hence difficulty in sending high-speed data over long distance in wireless systems. Thus 4G uses cell sizes $d \ll 1$Km for broadband access and use of Relays, micro-cells, can help.

Capacity of fading channels



Capacity of fading (wireless) channels

Wireless channels exhibits flat or frequency selective fading, which degrades capacity compared to AWGN channels.

Capacity of fading (wireless) channels

- The Shannon capacity of fading channel when channel state information (CSI) is available at the receiver, and with an average transmit power constraint $\bar{P}$ is given by:
$$C = \int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma$$
where $\gamma = \frac{\alpha^2\bar{P}}{N_0B}$, and α is the channel fading gain.
- Using Jensen's inequality
$\int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma \leq B\log_2(1+E[\gamma]) = B\log_2(1+\bar{\gamma})$
- $\bar{\gamma}$ is the average SNR in the fading channel, which is equivalent to the SNR of an AWGN channel. Thus the Shannon capacity of fading channel is less than the Shannon capacity of AWGN channel.

Capacity of fading (wireless) channels

Example: 2 (Example 4.2 in [1]))

Consider a flat fading channel with iid channel gain which can take three possible values: $\alpha_1 = 0.05$ with probability $p_1 = 0.1$, $\alpha_2 = 0.5$ with probability $p_2 = 0.5$, and $\alpha_3 = 1$ with probability $p_3 = 0.4$. The transmit power is 10mW, the noise PSD has $N_0 = 10^{-9}$ W/Hz, and the channel bandwidth is 30kHz. Assuming the receiver knows α_i but the transmitters does not, find the Shannon capacity of this channel and compare it with the capacity of AWGN channel with same average SNR.

Solution

- The channel has three possible received SNRs:
$\gamma_1 = \bar{P}\alpha_1^2/N_0B = 0.01(0.05)^2/(30000\times10^{-9}) = 0.8333 => -0.79\text{dB}$
$\gamma_2 = \bar{P}\alpha_2^2/N_0B = 0.01(0.5)^2/(30000\times10^{-9}) = 83.33 => 19.2\text{dB}$
$\gamma_3 = \bar{P}\alpha_3^2/N_0B = 0.01(1)^2/(30000\times10^{-9}) = 333.33 => 25\text{dB}$
- Probability of these SNRs are: $p_1 = 0.1, p_2 = 0.5, p_3 = 0.4$

Capacity of fading (wireless) channels

Solution (cont.)

- The Shannon capacity of the channel is given by: $C = \sum_i B \log_2(1+\gamma_i)p(\gamma_i)$
  $C = 30000\big(0.1\log_2 1.833 + 0.5\log_2 84.33 + 0.4\log_2 333.33\big) = 199.26$ kbps
- The average SNR of this wireless channel is
  $\bar{\gamma} = 0.1 \times 0.833 + 0.5 \times 83.33 + 0.4 \times 333.33 = 175.08 => 22.43$ dB
- The capacity of an AWGN channel with this SNR is
  $C = B\log_2(1+175.08) = 223.8$ kbps.
- Note that this rate is about 25 kbps larger than that of the flat fading channel with receiver CSI and the same average SNR.

Capacity of Fading channels with Outage

- Capacity with outage applies to slowly varying channels (where SNR γ is constant over a number of transmission bursts, then changes to a new value based on the fading distribution).
- The capacity in a burst of SNR γ is $C = B\log_2(1+\gamma)$.
- Since transmitter does not know the SNR value γ, it must fix transmission rate $R = C_{min} = B\log_2(1+\gamma_{min})$, where γ_{min} is the lowest SNR for which the channel is planned to be used successfully.
- If the instantaneous SNR at the receiver is below γ_{min}, transmitted data is incorrectly decoded, and receiver declares an outage and asks for re-transmission (ARQ).
- If the instantaneous SNR at the receiver is above γ_{min}, Transmitted data is correctly decoded.
- Probability of outage is $P_{out} = P[\gamma < \gamma_{min}]$.
- Throughput with outage $= (1-P_{out})B\log_2(1+\gamma_{min})$ bps.

γ_{min} is a design parameter based on acceptable outage.

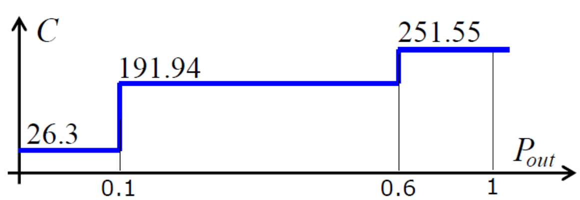
Example: 3 (Example 4.3 in [7])

Assume same channel in Example 2, with channel bandwidth of 30kHz and three possible received SNR values: $\gamma_1 = 0.8333$ with probability $p(\gamma_1) = 0.1$, $\gamma_2 = 83.33$ with probability $p(\gamma_2) = 0.5$, and $\gamma_3 = 333.33$ with probability $p(\gamma_3) = 0.4$. Find and sketch the capacity versus outage $0 \leq P_{out} < 1$ for this channel. Find the average rate correctly received (throughput) for outage probabilities.

Solution

- For discrete SNR levels, capacity versus outage is stair case
- For $\gamma_{min} = \gamma_1$, then $P_{out} = P[\gamma < \gamma_{min}] = 0$ is expected. Thus capacity achievable is $C = B\log_2(1+\gamma_{min}) = 30000\log_2(1.833) = 26.23$ kbps.
- For $\gamma_{min} = \gamma_2$, then $P_{out} = P[\gamma < \gamma_{min}] = 0.1$ then 10% outage expected and receiver must decode correctly > 90% of time. Thus capacity achievable is $C = B\log_2(1+\gamma_{min}) = 30000\log_2(84.33) = 191.94$ kbps.

- For $\gamma_{min} = \gamma_3$, then $P_{out} = P[\gamma < \gamma_{min}] = 0.6$ then 60% outage expected and receiver must decode correctly > 40% of time. Thus capacity achievable is $C = B\log_2(1+\gamma_{min}) = 30000\log_2(334.33) = 251.55$ kbps



- For $P_{out} < 0.1$, data sent are always correctly received since all SNR states support this rate, throughput=26.23kbps.
- For $P_{out} = 0.1$, data sent are correctly received with probability $(1 - P_{out})$, thus throughput=$(1 - P_{out})C = (1 - 0.1)191940 = 172.75$ kbps
- For $P_{out} = 0.6$, data sent are correctly received with probability $(1 - P_{out})$, thus throughput=$(1 - P_{out})C = (1 - 0.6)251550 = 125.78$ kbps
- A good engineering design for this channel may use $P_{out} = 0.1$, with ARQ.

Capacity of fading (wireless) channels

Channel Throughput

- Is the number of Data packets successfully transmitted over the wireless channel, bits/s.
- Thus, channel throughput is the actual data rate experienced by the user transmitting over the wireless channel. Telecom operators care mostly about the throughput.
- Designers plan with the channel capacity, which is the theoretical upper bound on the throughput.
- Throughput determined by measurements.
- **Example:** Wi-Fi: C=108Mbps, Throughput$= 60 - 100$ Mbps (depending on the device, channel location, & interference from other users).

Homework

Homework 6

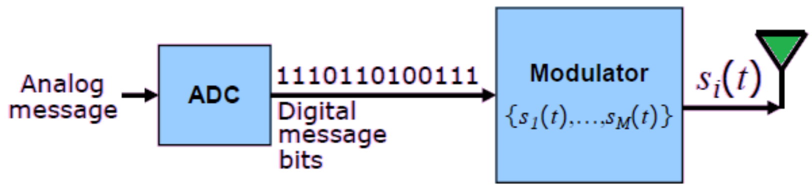
Consider a flat fading channel of bandwidth 20 MHz and where, for a fixed transmit power $\bar{P}$, the received SNR is one of six values: $\gamma_1 = 20dB$, $\gamma_2 = 15dB$, $\gamma_3 = 10dB$, $\gamma_4 = 5dB$, $\gamma_5 = 0dB$, $\gamma_6 = -5dB$. The probabilities associated with each state are $p_1 = p_6 = 0.1, p_2 = p_4 = 0.15$, and $p_3 = p_5 = 0.25$. Assume that only the receiver has CSI.

1. Find the Shannon capacity of this channel.
2. Plot the capacity versus outage for $0 \leq P_{out} < 1$
3. Find the maximum average rate that can be correctly received (Maximum C_{out})

Digital Modulation (Revision)

Principle of Operation

- Digital modulation techniques represent the message as a time sequence of symbols.
- While analog modulation varies a carrier signal physically with the message signal, digital modulation technique involves choosing a particular signal waveform $s_i(t)$, from a finite set of possible symbols $\{s_1(t), \ldots, s_M(t)\}$, to represent the information (message) bits at any time instant.
- In an M-level digital modulation, each symbol represents $k = log_2(M)$ bits of information per transmission (or channel use).



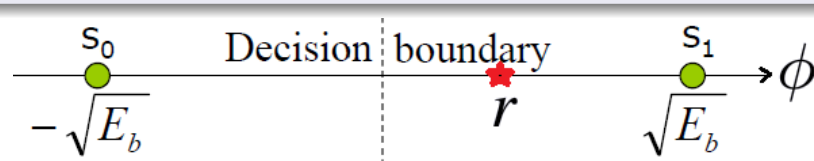
Digital Modulation (Revision)

Signal-to-noise Ratio (SNR)

- Let $s(t)$ be the transmitted signal and $r(t) = s(t) + n(t)$ be the received signal corrupted by AWGN, with power spectral density (PSD) of $N_0/2$.
- The Signal-to-noise ratio is given by $SNR = \frac{P_r}{N_0 B} = \frac{E_s}{N_0 B T_s} = \frac{E_b}{N_0 B T_b}$ where T_s and T_b are the symbol and bit duration respectively, and B is the bandwidth of the transmitted signal.
- For pulse shaping with $T_s = 1/B$ (e.g. raised cosine pulses with roll of factor, $\beta = 1$), $\Rightarrow SNR = E_s/N_0$.
- Approximate conversions:
  (1) $\gamma_b \approx \gamma_s/\log_2(M)$, where $\gamma_s = E_s/N_0$ and $\gamma_b = E_b/N_0$ are SNR per symbol and SNR per bit respectively
  (2) $p_b(error) \approx p_s(error)/\log_2(M)$, where $p_s(error)$ & $p_b(error)$ are symbol and bit error probabilities.

(1) Binary Phase Shift Keying (BPSK)

- Depending on the input bits, a BPSK modulator chooses one of two sets of signals:
$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_c) \qquad 0 \le t \le T_b$
$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi + \theta_c) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_c) \qquad 0 \le t \le T_b$
- It select $S_1(t)$ when input bit is "1" and select $S_2(t)$ when input bit is "0".
- $S_1(t)$ and $S_2(t)$ have same amplitude, but shifted in phase by π.
- In practice the transmitted power is selected such that the carrier has unit energy measured over one bit duration such that the carrier is given by
$\phi(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t + \theta_c) \qquad 0 \le t \le T_b$
- Then $S_{BPSK}(t) = \pm\sqrt{E_b}\phi(t)$.

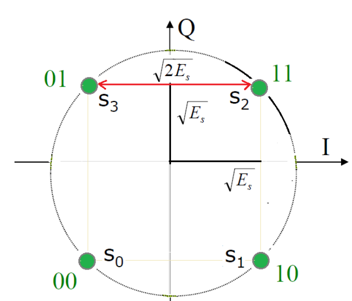


(2) Quadrature Phase Shift Keying (QPSK)

- QPSK (4-PSK) modulator uses each symbol to represent two bits of information at a time. Thus QPSK has twice bandwidth efficiency of BPSK.
- In QPSK, the phase of a constant amplitude carrier is switched between one of four equally spaced values:

$$S_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} \cos(2\pi f_c t + (2i-1)\frac{\pi}{4}) \quad 0 \le t \le T_s$$

Where $i = 1, 2, 3, 4$
T_s & E_s is the symbol duration & symbol energy
- Using trigonometric identities, we have:
$S_{QPSK}(t) = \sqrt{\frac{2E_s}{T_s}} \cos((2i-1)\frac{\pi}{4}) \cos(2\pi f_c t) + \sqrt{\frac{2E_s}{T_s}} \sin((2i-1)\frac{\pi}{4}) \sin(2\pi f_c t)$



$S_{QPSK}(t) = m_I(t)\phi_1(t) + m_Q(t)\phi_2(2)$
where
$\phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t)$
$\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t)$
$m_I(t) = \pm\sqrt{E_s}$
$m_Q(t) = \pm\sqrt{E_s}$

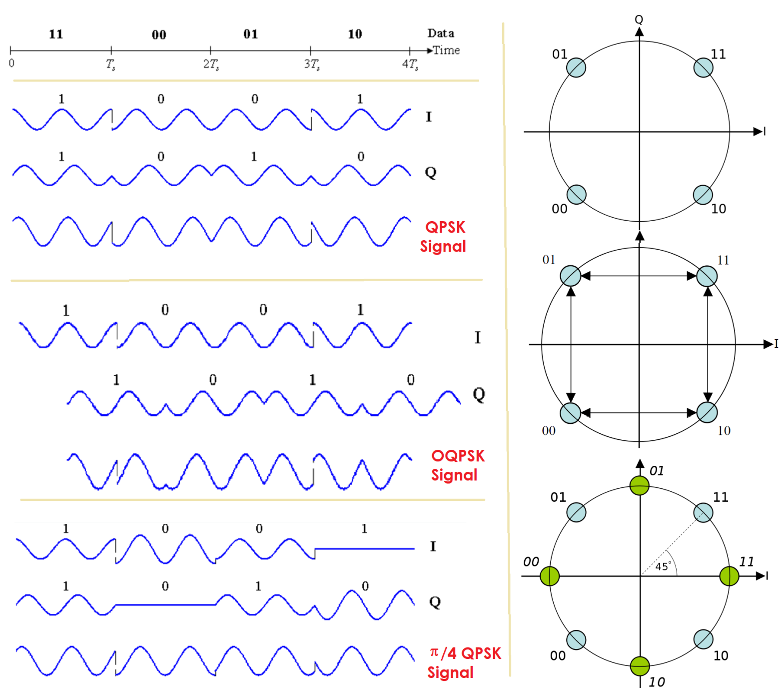
(3) Offset Quadrature Phase Shift Keying (OQPSK)

- In QPSK, $m_I(t)$ and $m_Q(t)$ are aligned with phase transition of the pulses occurring every $T_s = 2T_b$, making it possible to have 180^0 phase change when both values of $m_I(t)$ and $m_Q(t)$ change.
- In OQPSK, the transition instant of $m_I(t)$ and $m_Q(t)$ are offset by T_b such that transitions occur every T_b seconds, and only one of the two bit streams $m_I(t), m_Q(t)$ can change value any transition.
- Thus maximum phase change in OQPSK is $\pm 90^0$ whereas maximum phase transitions in QPSK is 180^0.

(4) $\pi/4$-QPSK

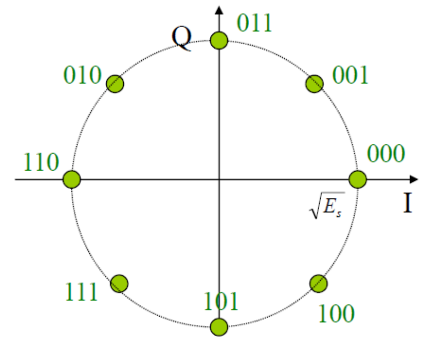
- QPSK looses its constant envelope when pulse shaped, because a shift of π causes signal envelope to go zero momentarily, since $\{-A(t-\tau) + A(t) \approx 0\}$. Thus, to avoid distortion, QPSK can only be amplified using inefficient linear high-power amplifiers (HPA), after shaping.
- $\pi/4$-QPSK: avoids this by rotating QPSK constellation anticlockwise by $\pi/4$-radians, so that maximum phase transition becomes $\pm 135^0$

QPSK vs. OQPSK vs. $\pi/4$-QPSK



(5) M-ary PSK (M-PSK)

The phase of a constant amplitude carrier is switched between one of M equally spaced values. where each symbol represents $k = \log_2 M$ bits of information. (e.g. 8-PSK)

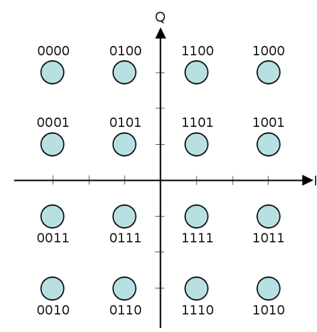


(5) M-ary QAM (M-QAM)

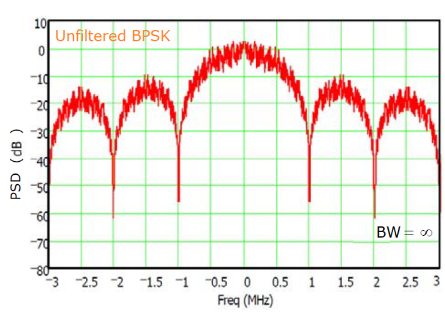
Both the amplitude and phase of the carrier are varied in order to obtain different sets of M-ary QAM signals. The general form of an M-ary QAM signal can be defined as:

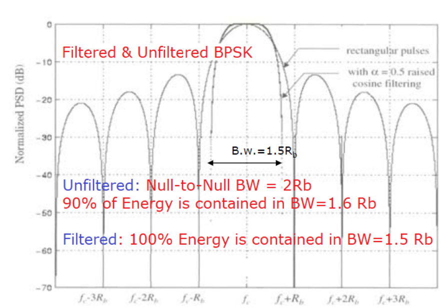
$$S_{QAM} = \sqrt{\frac{2E_{min}}{T_s}} a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_{min}}{T_s}} b_i \sin(2\pi f_c t)$$

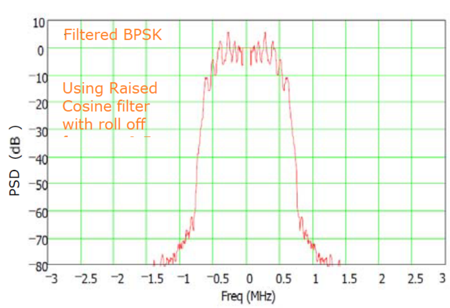
for $i = 1, ..., M$ where E_{min} = energy of the signal with the lowest amplitude, and a_i and b_i are independent integers.
(e.g. 16-QAM)

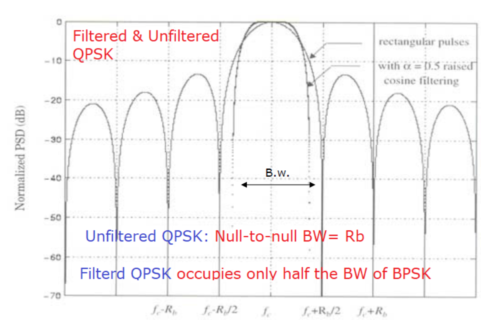


Spectral Analysis of filtered/unfiltered BPSK and QPSK









M-PSK versus M-QAM

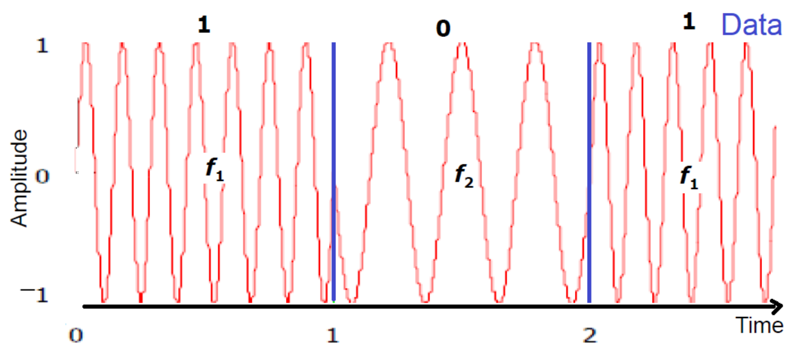
- PSK keeps the message in the phase of the carrier.
- QAM keeps the message both in the amplitude and the phase (higher data rate).
- High Power Amplifiers (HPA) distort signal amplitude, thus QAM SER performance is severely affected compared to PSKs.
- 2G/3G digital cellular thus deployed variants of M-PSK, $M \leq 4$.
- However high data rate signaling is required in 4G systems, making QAM a compelling natural choice in broadband wireless systems.
- QAM is deployed in Wi-Fi, Wi-MAX, and 4G-LTE.

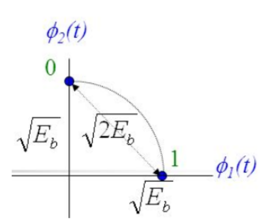
Constant Envelope Modulation:

- It allows the efficient use of HPA and it includes:
  (1) Binary Frequency Shift Keying (BFSK)
  (2) M-ary FSK
  (3) Continuous-Phase Frequency Shift Keying (CPFSK)
  (4) Minimum-Shift Keying (MSK) and Gaussian MSK (GMSK)

BFSK

- In general BFSK signal is generated by switching the frequency of a constant amplitude carrier between two values f_1 & f_2, to represent binary "1" and binary "0".
  $S_{BFSK}(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_1 t) \quad 0 \leq t \leq T_b \Rightarrow (binary"1")$
  $S_{BFSK}(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_2 t) \quad 0 \leq t \leq T_b \Rightarrow (binary"0")$
- Its drawback is that the phase discontinuity at bit transitions will cause spectral spreading.





Continuous-Phase Frequency Shift Keying (CPFSK)

- In CPFSK, f_1 and f_2 are chosen to ensure that there is no phase discontinuity at bit transitions, i.e. when switching f_1 & f_2, to represent bit "1" and bit "0".
- This can be achieved by choosing the frequencies, f_1 and f_2 such that they differ from each other by an amount equal to the reciprocal of the bit duration T_b, i.e. $f_1 - f_2 = kR_b$ (switching rate)

Minimum-Shift Keying (MSK)

- In FSK, overall difference Δf in the transmitted frequency from symbol '0' to symbol '1', or vice-versa, is matched to bit-rate of the incoming data stream.
- Minimum shift keying (MSK) uses the minimum value of Δf possible.
- Thus MSK uses the value of Δf given by: $\Delta f = f_1 - f_2 = \frac{1}{2T_b}$

  $f_c = \frac{f_1+f_2}{2}$ is the frequency of un-modulated carrier

  $f_1 = f_c + \frac{\Delta f}{2}$

  $f_2 = f_c - \frac{\Delta f}{2}$

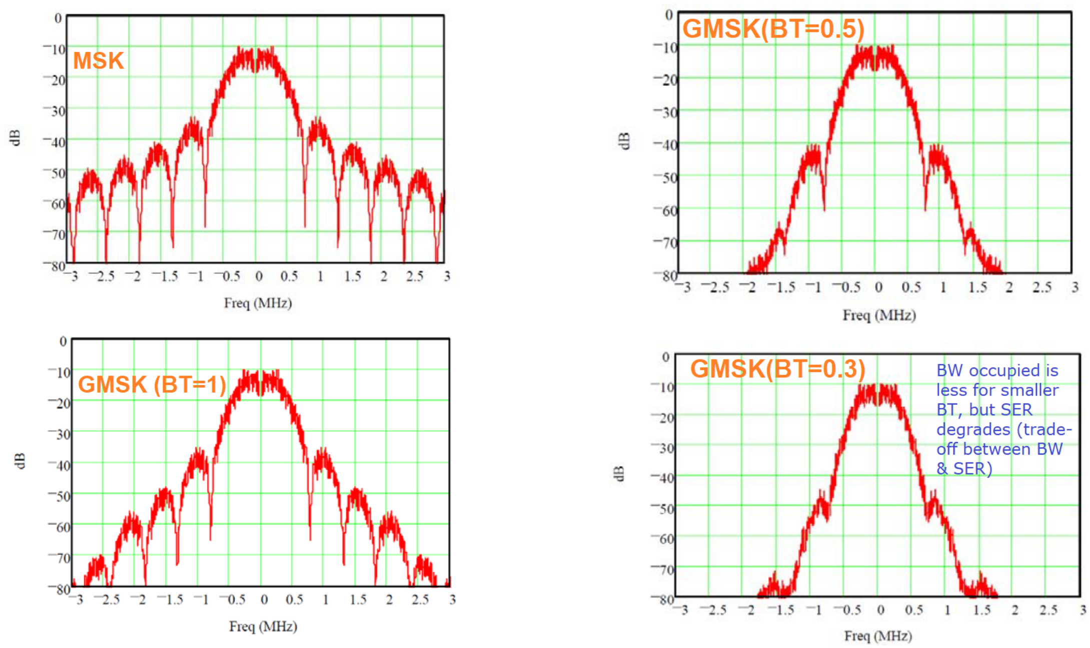
GMSK

- GMSK is an MSK scheme in which a pre-modulation low pass filter with Gaussian characteristics is employed, to smoothen phase transitions of MSK.
- Baseband filtering is made such that it gives the required RF spectrum.
- It achieves better uniform envelope, with superior spectral containment. Uniform envelop allows efficient nonlinear HPAs (class C) to be used.
- Gaussian filtering introduces ISI (represented by the side lobes) in the signal. However performance loss due to this is negligible as long as the ISI introduced by the GMSK is less than that produced by the mobile channel.
- GMSK is used in GSM with $BT = 0.3$. GMSK is spectrally tighter than MSK.

Occupied RF BW containing a given % of power for GMSK & MSK.

BT	90%	99%	99.9%	99.99%
0.2 GMSK	0.52 R_b	0.79 R_b	0.99 R_b	1.22 R_b
0.25 GMSK	0.57 R_b	0.86 R_b	1.09 R_b	1.37 R_b
0.5 GMSK	0.69 R_b	1.04 R_b	1.33 R_b	2.08 R_b
MSK	0.78 R_b	1.20 R_b	2.76 R_b	6.00 R_b

Spectral Analysis of MSK and GMSK



Example: 4

Find the 3-dB bandwidth for a Gaussian low pass filter used to produce 0.25 GMSK with a channel data rate of $R_b = 270$ kbps. What is the 90% power bandwidth in the RF channel?.

Solution

- $T_b = 1/R_b = 1/(270 \times 10^3) = 3.7\mu s$
- Solving for B, where BT= 0.25, we have:
  $B = 0.25/T_b = 0.25/(3.7 \times 10^{-6}) = 67.567$ kHz.
- Using the Table, The 90% power of GMSK is contained in the RF BW of $0.57R_b$.
- Hence RF $BW = 0.57 \times 270 \times 10^3 = 153.9$ kHz.

BER Performance in AWGN channels

BER of coherent BPSK: $P_{e,AWGN} = Q(\frac{d_{min}}{\sqrt{2N_0}}) = Q(\sqrt{\frac{2E_b}{N_0}})$

BER of coherent QPSK: $P_{e,AWGN} = Q(\frac{d_{min}}{\sqrt{2N_0}}) = Q(\sqrt{\frac{2E_b}{N_0}})$

BER of coherent BFSK: $P_{e,AWGN} = Q(\frac{d_{min}}{\sqrt{2N_0}}) = Q(\sqrt{\frac{E_b}{N_0}})$

BER of coherent GMSK: $P_{e,AWGN} = Q(\sqrt{\frac{2\gamma E_b}{N_0}})$ where γ is constant related to BT as

$$\gamma = \begin{cases} 0.68 & \text{for GMSK with } BT = 0.25 \\ 0.85 & \text{for GMSK with } BT = \infty \end{cases}$$

BER for MSK is 1 dB better than GMSK with $BT = 0.25$

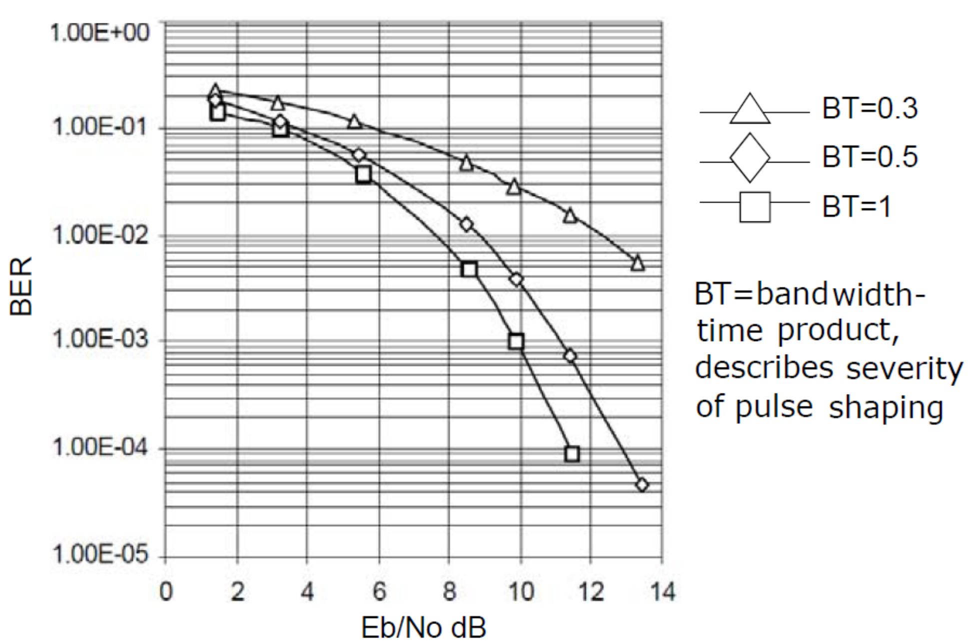
BER of noncoherent BFSK: $P_{e,AWGN} = 0.5e^{-\frac{E_b}{N_0}}$

SER for coherent M-PSK: "$d_{min} = 2\sqrt{E_s}\sin(\frac{\pi}{M})$"

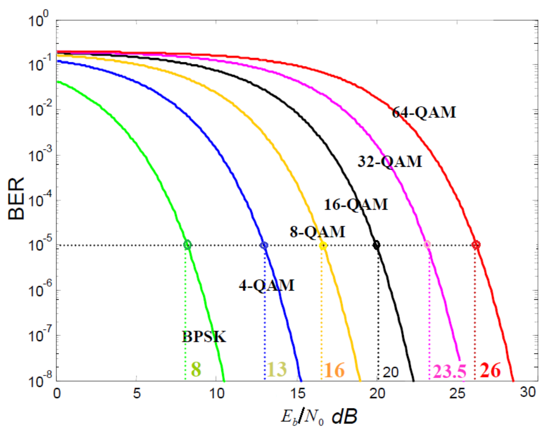
$P_{e,MPSK} \leq 2Q\left(\sqrt{\frac{2E_b \log_2 M}{N_0}}\sin(\frac{\pi}{M})\right)$ and $P_{e,MPSK} \approx 2Q\left(\sqrt{\frac{4E_s}{N_0}}\sin(\frac{\pi}{2M})\right)$ for $M \geq 4$

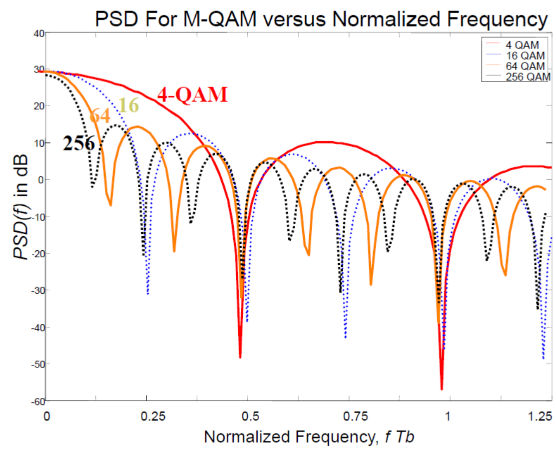
SER for coherent M-QAM: $P_{e,MQAM} \approx 2(1 - \frac{1}{\sqrt{M}})Q\left(\sqrt{\frac{3E_{av}}{(M-1)N_0}}\right)$

BER Performance of MSK and GMSK in AWGN channels



BER Performance of M-QAM in AWGN channels





Bandwidth and Power Efficiency of Digital modulations

- Bandwidth efficiency, $\eta = \frac{R_b}{BW}$: A measure of how a modulation scheme makes use of the available bandwidth.
- First null bandwidth for M-QAM (and M-PSK as well) decreases as M increases. That is, bandwidth efficiency increases as the value of M increases.
- Power efficiency, $\frac{E_b}{N_0}$:How much signal power (energy per bit) is needed to reach a given BER performance. As M increases, the constellation is more densely packed, increasing the probability of error and thus higher signal power must be transmitted to maintain acceptable error rate. That is power efficiency reduces as M increases.

M	2	4	8	16	32	64
η_B	0.5	1	1.5	2	2.5	3
$\frac{E_b}{N_0}$ for $BER = 10^{-5}$	8	13	16	20	23.5	26

Digital Modulation in current Cellular Systems

Standard	Principle region of operation	Access Method	Modulation Scheme:
IS-54 (USDC)	North America	TDMA	π/4-QPSK (α=0.35)
GSM	World-wide	TDMA	GMSK (BT=0.3)
IS-95(CDMA)	North America	CDMA	QPSK & OQPSK*
PDC	Japan	TDMA	π/4-QPSK (α=0.5)
WiMAX / LTE	World-wide	OFDMA, SC-FDMA+	QAM

*Offset QPSK, +Single-carrier FDMA

Digital Modulation Performance in Fading

- Let $P_{e,AWGN}(\gamma_s)$ denote the probability of error performance of a digital modulation over AWGN channel, at a particular SNR γ.
- Then, the probability of error performance of the system in slow flat–fading channels can be calculated as: $P_{e,Fading}(\gamma_s) = \int_0^\infty P_{e,AWGN}(\gamma_s)f(\gamma)d\gamma$ where $\gamma_s = \frac{\alpha^2 E_s}{N_0}$ is the SNR in fading channel with fading gain α, E_s is the average symbol energy while N_0 is the noise power.
- $f(\gamma)$ is the pdf of γ due to fading.

Non-Coherent BFSK

- $P_{e,AWGN}(\gamma_s) = 0.5e^{-\frac{E_b}{N_0}}$
- $P_{e,Fading}(\gamma_s) = \int_0^\infty 0.5e^{-\frac{E_b}{N_0}} f(\gamma)d\gamma = \int_0^\infty 0.5e^{-\frac{E_b}{N_0}} [\frac{1}{\bar{\gamma}}e^{-\frac{\gamma}{\bar{\gamma}}}]d\gamma = \frac{1}{2[\bar{\gamma}+1]}$

Note: $\bar{\gamma} = E[\gamma]$

Digital Modulation Performance in Fading

Binary PSK in Rayleigh

$P_e = 0.5(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}) \approx \frac{1}{4\bar{\gamma}}$

Binary FSK in Rayleigh

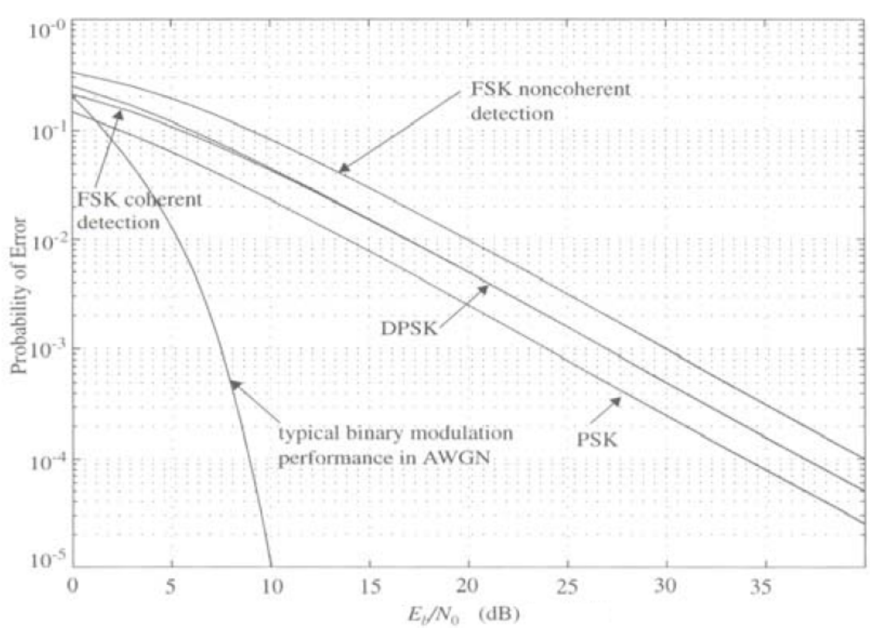
$P_e = 0.5(1 - \sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}}) \approx \frac{1}{4\bar{\gamma}}$

For Differential PSK (DPSK)

For DPSK, a non-coherent form of PSK which avoids the need for coherent reference signal at receiver. $P_e = \frac{1}{2(1+\bar{\gamma})} \approx \frac{1}{2\bar{\gamma}}$

Note: roughly 3-dB power penalty using DPSK compared to PSK.

Digital Modulation Performance in Fading



Homework

Homework 7

Plot analytically and using MATLAB simulations the BER versus E_b/N_0 performance for BPSK, DPSK and QPSK in additive white Gaussian noise. List advantages and disadvantages of each modulation method from the mobile communication standpoint.

Equalization to be next...